Algebraic Aspects of the Russian Hash Standard GOST R 34.11-2012

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Agenda

Basic Operations and Functions

GOST R 34.11-2012 (Stribog) is based on SP-network block cipher with block and key length equal 512 bits

- SubBytes (S): nonlinear bijective mapping.
- Transposition (P): byte permutation.
- MixColumns (L): linear transformation.
- AddRoundKey (X): addition with the round key using bitwise XOR.

Other basic functions

- $\boxplus$: addition modulo 2^{512}.
- $MSB_s(A)$: getting s most significant bits of vector A.
- $A||B$: concatenation of two vectors A and B.

State Representation

Grøstl

a_0	a_8	a_{16}	a_{24}	a_{32}	a_{40}	a_{48}	a_{56}
a_1	a_9	a_{17}	a_{25}	a_{33}	a_{41}	a_{49}	a_{57}
a_2	a_{10}	a_{18}	a_{26}	a_{34}	a_{42}	a_{50}	a_{58}
a_3	a_{11}	a_{19}	a_{27}	a_{35}	a_{43}	a_{51}	a_{59}
a_4	a_{12}	a_{20}	a_{28}	a_{36}	a_{44}	a_{52}	a_{60}
a_5	a_{13}	a_{21}	a_{29}	a_{37}	a_{45}	a_{53}	a_{61}
a_6	a_{14}	a_{22}	a_{30}	a_{38}	a_{46}	a_{54}	a_{62}
a_7	a_{15}	a_{23}	a_{31}	a_{39}	a_{47}	a_{55}	a_{63}

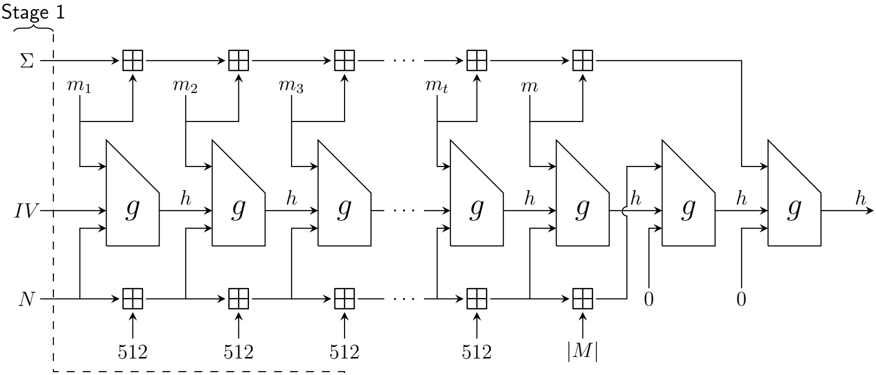
Stribog

a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7
a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
a_{16}	a_{17}	a_{18}	a_{19}	a_{20}	a_{21}	a_{22}	a_{23}
a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}	a_{31}
a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	a_{37}	a_{38}	a_{39}
a_{40}	a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}	a_{47}
a_{48}	a_{49}	a_{50}	a_{51}	a_{52}	a_{53}	a_{54}	a_{55}
a_{56}	a_{57}	a_{58}	a_{59}	a_{60}	a_{61}	a_{62}	a_{63}

$$A = a_0||a_1|| \ldots ||a_{63}$$

Agenda

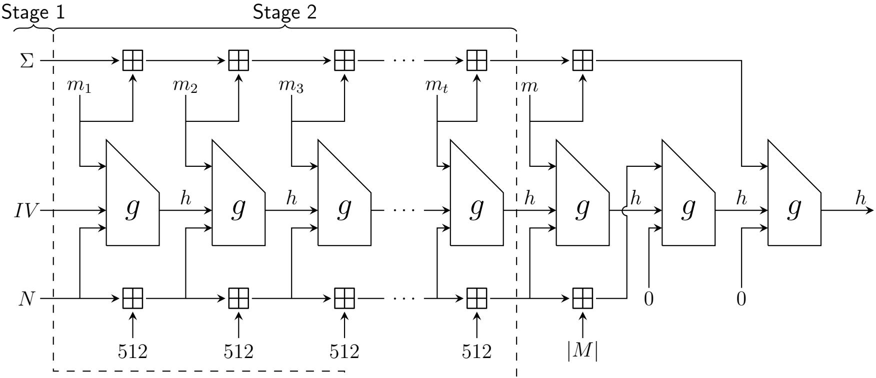
Hash Function Stribog. Stage 1



$\Sigma = N = 0^{512}$

$$IV = \begin{cases} 0, & \text{Stribog-512} \\ (00000001)^{64}, & \text{Stribog-256} \end{cases}$$

Hash Function Stribog. Stage 2

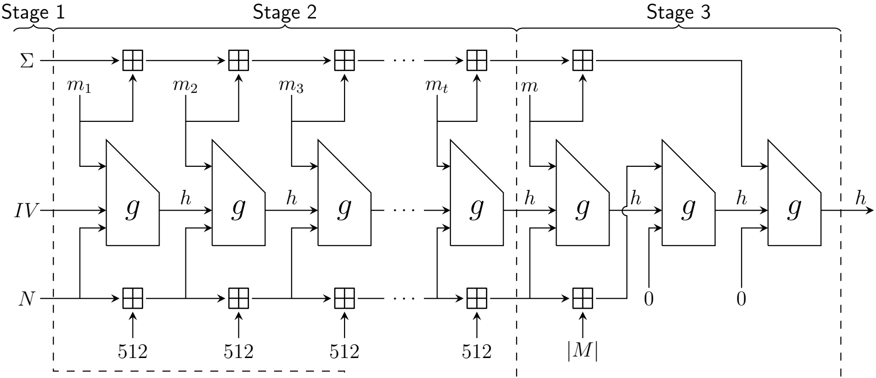


$\Sigma = N = 0^{512}$

$IV = \begin{cases} 0, & \text{Stribog-512} \\ (00000001)^{64}, & \text{Stribog-256} \end{cases}$

$t = \left\lfloor \frac{|M|}{512} \right\rfloor$

Hash Function Stribog. Stage 3



$\Sigma = N = 0^{512}$

$IV = \begin{cases} 0, & \text{Stribog-512} \\ (00000001)^{64}, & \text{Stribog-256} \end{cases}$

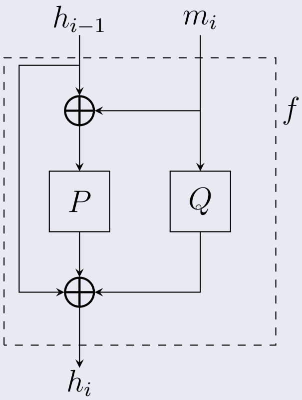
$t = \left\lfloor \frac{|M|}{512} \right\rfloor$

$m = 0^{512-|M|}||1||M$

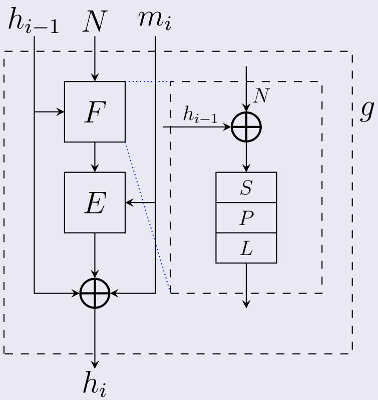
$H = \begin{cases} h, & \text{Stribog-512} \\ MSB_{256}(h), & \text{Stribog-256} \end{cases}$

Construction of the Compression Function g

Grøstl

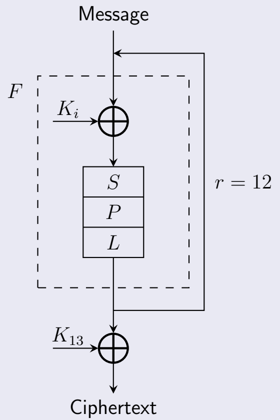


Stribog

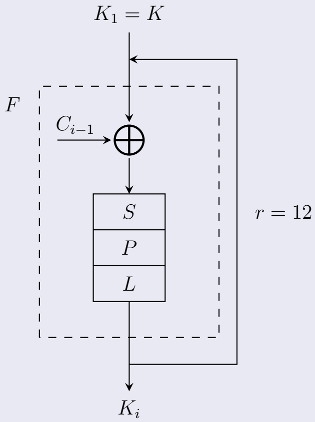


Representation of E

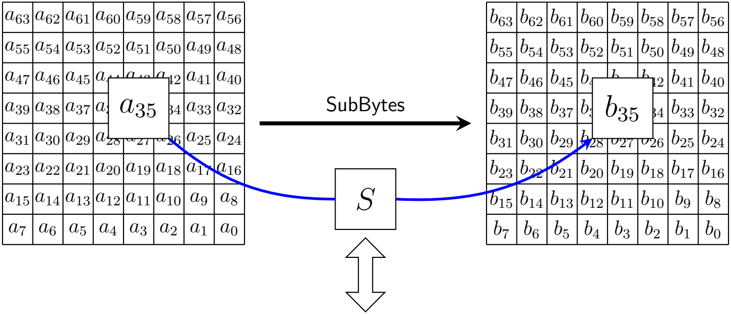
Block Cipher of Stribog



Key Schedule



SubBytes (S)



$$b_i = \text{S-box}(a_i)$$

Transposition (P)

Transposition transformation has a form

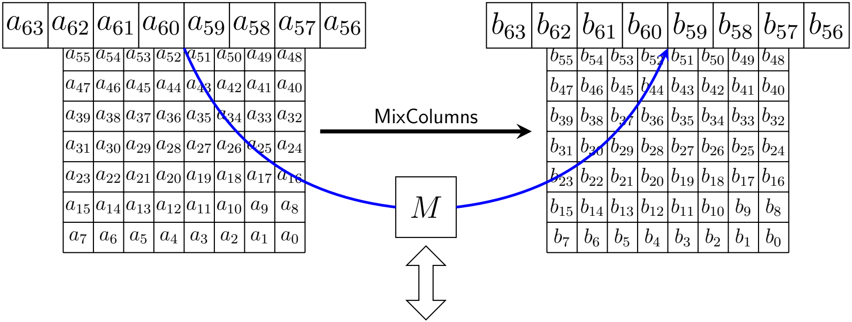
a_{63}	a_{62}	a_{61}	a_{60}	a_{59}	a_{58}	a_{57}	a_{56}
a_{55}	a_{54}	a_{53}	a_{52}	a_{51}	a_{50}	a_{49}	a_{48}
a_{47}	a_{46}	a_{45}	a_{44}	a_{43}	a_{42}	a_{41}	a_{40}
a_{39}	a_{38}	a_{37}	a_{36}	a_{35}	a_{34}	a_{33}	a_{32}
a_{31}	a_{30}	a_{29}	a_{28}	a_{27}	a_{26}	a_{25}	a_{24}
a_{23}	a_{22}	a_{21}	a_{20}	a_{19}	a_{18}	a_{17}	a_{16}
a_{15}	a_{14}	a_{13}	a_{12}	a_{11}	a_{10}	a_9	a_8
a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0

Transpose →

a_{63}	a_{55}	a_{47}	a_{39}	a_{31}	a_{23}	a_{15}	a_7
a_{62}	a_{54}	a_{46}	a_{38}	a_{30}	a_{22}	a_{14}	a_6
a_{61}	a_{53}	a_{45}	a_{37}	a_{29}	a_{21}	a_{13}	a_5
a_{60}	a_{52}	a_{44}	a_{36}	a_{28}	a_{20}	a_{12}	a_4
a_{59}	a_{51}	a_{43}	a_{35}	a_{27}	a_{19}	a_{11}	a_3
a_{58}	a_{50}	a_{42}	a_{34}	a_{26}	a_{18}	a_{10}	a_2
a_{57}	a_{49}	a_{41}	a_{33}	a_{25}	a_{17}	a_9	a_1
a_{56}	a_{48}	a_{40}	a_{32}	a_{24}	a_{16}	a_8	a_0

MixColumns (L)

MixColumns transformation has a form



Multiplying the vector by the constant 64×64 matrix M over $\mathbb{F}_2$

$$B = A \cdot M$$

Agenda

Motivation

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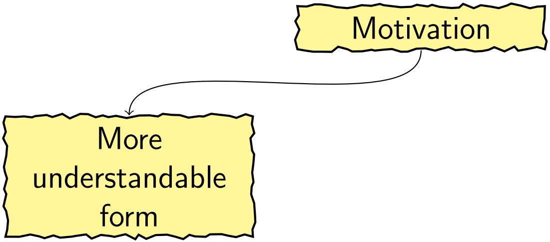

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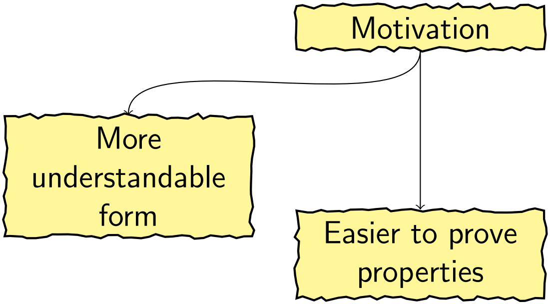
Motivation
More understandable form
Easier to prove properties

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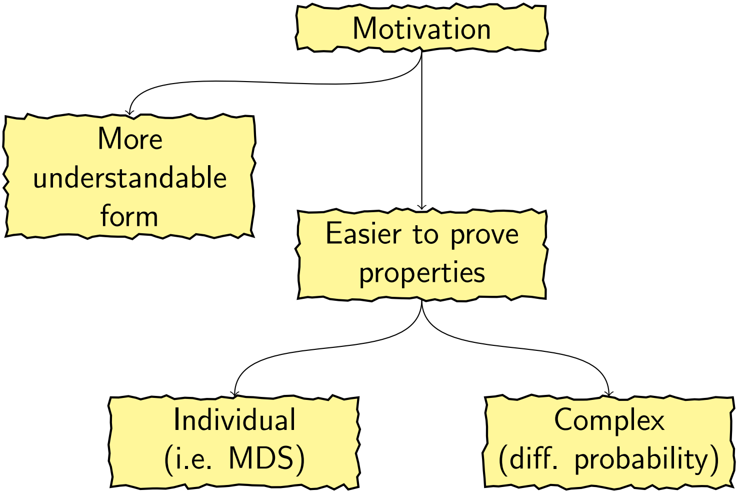
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More understandable form
Easier to prove properties
Individual (i.e. MDS)
Complex (diff. probability)

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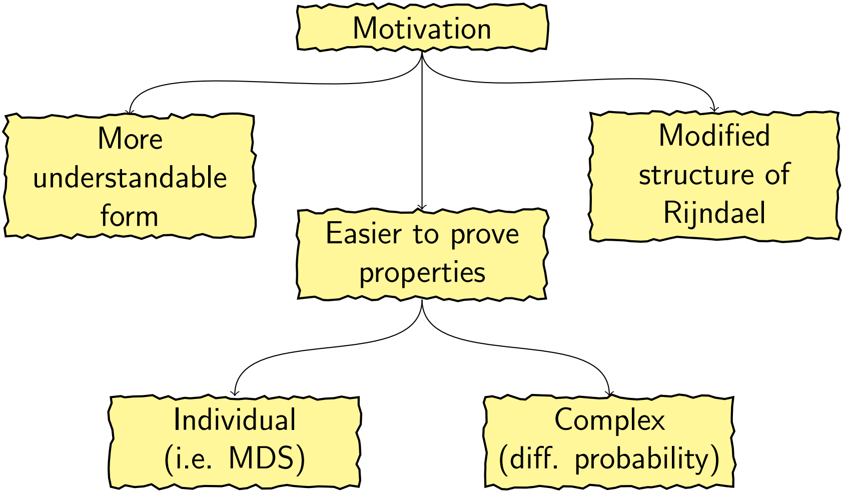
Motivation
More understandable form
Easier to prove properties
Modified structure of Rijndael
Individual (i.e. MDS)
Complex (diff. probability)

State Representation

Alternative representation

- Reverse input bits
- AES-like transformations (state as in Grøstl)
- Reverse output bits

$$\begin{array}{ccc} B_0, B_1, \ldots, B_{63} & & B'_0, B'_1, \ldots, B'_{63} \\ \updownarrow & & \updownarrow \\ b_0, b_1, \ldots, b_{511} & \xleftrightarrow{\text{ReverseBits}} & b_{511}, b_{510}, \ldots, b_0 \end{array}$$

Transposition and SubBytes Operations

- Transposition is invariant operation.
- Substitution has the form $F(x) = D \circ G \circ D(x)$ for linearized polynomial $D : \mathbb{F}_{2^n} \mapsto \mathbb{F}_{2^n}$.

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	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**A**	**B**	**C**	**D**	**E**	**F**
0	3F	FB	D7	E0	9F	E5	A8	04	97	07	AD	87	A0	B5	4C	9A
1	DF	EB	4F	0C	81	58	CF	D3	E8	3B	FD	B1	60	31	B6	8B
2	F3	7C	57	61	47	78	08	B4	C9	5E	10	32	C7	E4	FF	67
3	C4	3E	BF	11	D1	26	B9	7D	28	72	39	53	FE	96	C3	9C
4	BB	24	34	CD	A6	06	69	E6	0F	37	70	C1	40	62	98	2E
5	5F	6B	16	D6	3C	1C	1E	A4	8F	14	C8	55	B7	A5	63	F5
6	8C	C2	12	B8	F7	46	59	90	99	0D	6E	1F	F1	AA	51	2D
7	20	9D	73	E7	71	64	4D	36	FA	50	BA	A1	CB	A9	B0	C6
8	77	AF	2C	1A	18	E9	85	8E	EE	F0	0E	D8	21	A2	AE	65
9	23	9E	54	EC	38	1D	89	D9	6C	17	4E	CA	D0	C5	2A	66
A	76	15	13	35	3A	00	DE	D4	74	29	30	FC	56	7A	AC	2F
B	A3	44	5C	9B	80	F9	79	A7	B3	CC	ED	1B	2B	AB	BD	D2
C	88	95	8A	02	5A	CE	94	25	DB	7B	6A	92	75	49	BC	4B
D	5B	6F	45	27	42	41	F6	0B	DD	0A	E2	09	19	BE	01	43
E	68	93	D5	EF	84	22	E3	DA	5D	3D	48	7F	05	F4	7E	03
F	B2	C0	33	91	F2	82	8D	4A	83	52	E1	86	F8	DC	EA	6D

Table : The Substitution F for AES-like Description

Representation of MixColumns (1/4)

The are exist at least three forms:

1. representation over $\mathbb{F}_{2^n}$
2. representation over $\mathbb{F}_2$
    1. matrix form
    2. system of equations

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$$\mathbb{F}_{2^n} \xrightarrow{\;easy\;} \mathbb{F}_2$$

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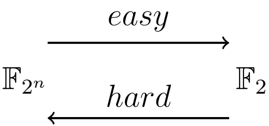

Representation of MixColumns (2/4)

Let $L : \mathbb{F}_{2^n} \mapsto \mathbb{F}_{2^n}$ be a linear function of the form

$$L(x) = \sum_{i=0}^{n-1} \delta_i x^{2^i}, \quad \delta_i \in \mathbb{F}_{2^n}.$$

Proposition

Any linear function $L : \mathbb{F}_{2^n} \mapsto \mathbb{F}_{2^m}$ can be converted to a matrix with the complexity $O(n)$.

$$L(x) = \delta x, \quad \delta_i = 0, \text{ for } 1 \leq i \leq n-1.$$

Representation of MixColumns (3/4)

Any multiplication mapping $\mathbb{F}_{2^n} \mapsto \mathbb{F}_{2^n}$ is a linear transformation of a vector space over $\mathbb{F}_2$ for specified basis.

Multiplication by arbitrary $\delta \in \mathbb{F}_{2^8}$ can be represented as multiplication by a matrix

$$\delta x = \begin{pmatrix} k_{0,0} & \cdots & k_{0,7} \\ k_{1,0} & \cdots & k_{1,7} \\ \vdots & \ddots & \vdots \\ k_{7,0} & \cdots & k_{7,7} \end{pmatrix} \cdot \begin{pmatrix} x_0 \\ x_1 \\ \ldots \\ x_7 \end{pmatrix}$$

where $x_i, k_{j,s} \in \mathbb{F}_2$.

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where $x_i, k_{j,s} \in \mathbb{F}_2$.

Representation of MixColumns (4/4)

The main steps of proposed algorithm for obtaining MDS matrix over $\mathbb{F}_{2^8}$ from 64×64 matrix over $\mathbb{F}_2$

1. for every irreducible polynomial (30)
   1. convert each of 8×8 submatrices to the element of the filed
   2. check MDS property of the resulting matrix

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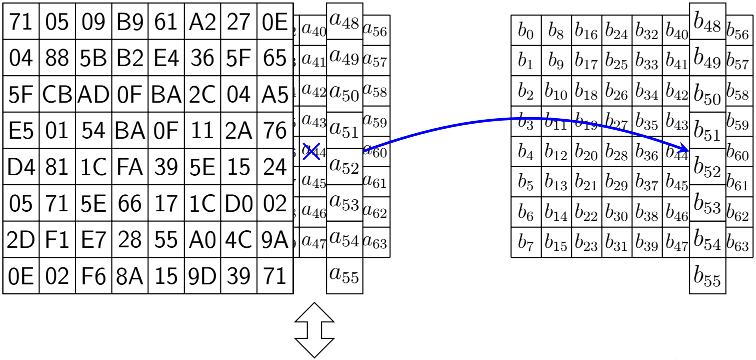
Hint

It is necessary to transpose matrix of Stribog before applying the algorithm.

MixColumns

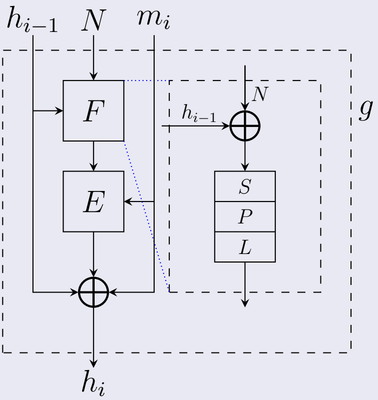


Multiplying the vector by the constant 8×8 matrix G over $\mathbb{F}_{2^8}$ with the primitive polynomial $f(x) = x^8 + x^6 + x^5 + x^4 + 1$

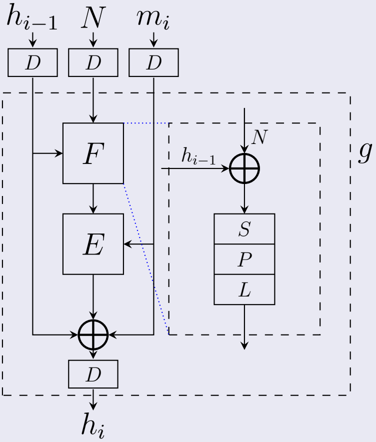
$$B = G \cdot A$$

AES-like Form of Compression Function

Original Function



Modified Function



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- Performance of GOST R 34.11-2012 is based on the message length.
- Proposed method has many application fields.
- More details on https://github.com/okazymyrov